Differentiation & Derivative

Definitions from the Oxford concise dictionary of Mathematics.

differentiation The process of obtaining the *derived function f' from the function f, where $f'(x)$ is the *derivative of f at x. *See* FIRST PRINCIPLES. The derivatives of certain common functions are given in the Table of derivatives (Appendix 2), and from these many other functions can be differentiated using the following rules of differentiation:

(i) If $h(x)=kf(x)$ for all x, where k is a constant, then $h'(x)=kf'(x)$.

(ii) If $h(x)=f(x)+g(x)$ for all x, then $h'(x)=f'(x)+g'(x)$.

(iii) The product rule: If $h(x)=f(x)g(x)$ for all x, then

$$h'(x)=f(x)g'(x)+f'(x)g(x).$$

(iv) The reciprocal rule: If $h(x)=1/f(x)$ and $f(x)\neq 0$ for all x, then

$$h'(x)=-\frac{f'(x)}{(f(x))^2}.$$

(v) The quotient rule: If $h(x)=f(x)/g(x)$ and $g(x)\neq 0$ for all x, then

$$h'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}.$$

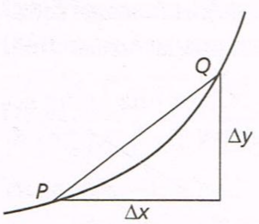
(vi) The *chain rule: If $h(x)=(f\circ g)(x)=f(g(x))$ for all x, then

$$h'(x)=f'(g(x))g'(x).$$

derivative For the *real function f, if $(f(a+h)-f(a))/h$ has a *limit as $h\to 0$, this limit is the derivative of f at a and is denoted by $f'(a)$. (The term 'derivative' may also be used loosely for the *derived function.)

Consider the graph $y=f(x)$. If (x, y) are the coordinates of a general point P on the graph, and $(x+\Delta x, y+\Delta y)$ are those of a nearby point Q on the graph, it can be said that a change Δx in x produces a change Δy in y. The quotient $\Delta y/\Delta x$ is the gradient of the chord PQ. Also, $\Delta y=f(x+\Delta x)-f(x)$. So the derivative of f at x is the limit of the quotient $\Delta y/\Delta x$ as $\Delta x\to 0$. This limit can be denoted by dy/dx, which is thus an alternative notation for $f'(x)$. The notation y' is also used.

The derivative $f'(x)$ may be denoted by $(d/dx)(\ )$, where the brackets contain a formula for $f(x)$. Some authors use the notation df/dx. The derivative $f'(a)$ gives the gradient of the curve $y=f(x)$, and hence the gradient of the tangent to the curve, at the point given by $x=a$. Suppose now, with a different notation, that x is a function of t, where t is some measurement of time. Then the derivative dx/dt, which is the *rate of change of x with respect to t, may be denoted by $\dot{x}$. The derivatives of



certain common functions are given in the Table of derivatives (Appendix 2). *See also* DIFFERENTIATION, LEFT AND RIGHT DERIVATIVE, HIGHER DERIVATIVE and PARTIAL DERIVATIVE.