Mutually Exclusive Events

Mutually Exclusive: can't happen at the same time.

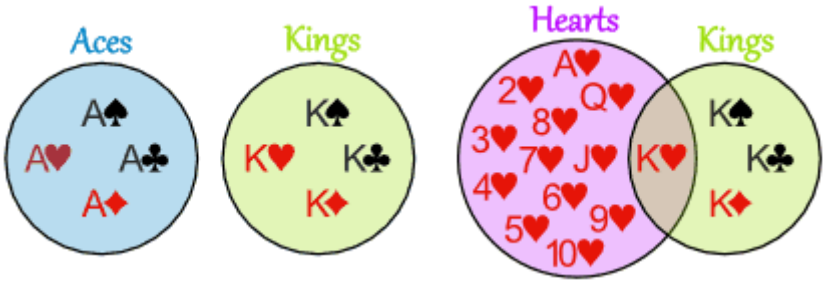
Examples:

- Turning left and turning right are Mutually Exclusive (you can't do both at the same time)
- Tossing a coin: Heads and Tails are Mutually Exclusive
- Cards: Kings and Aces are Mutually Exclusive

What is **not** Mutually Exclusive:

- Turning left and scratching your head can happen at the same time
- Kings and Hearts, because we can have a King of Hearts!

Like here:



Aces and Kings are **Mutually Exclusive** (can't be both)

Hearts and Kings are **not** Mutually Exclusive (can be both)

Probability

Let's look at the probabilities of Mutually Exclusive events. But first, a definition:

Probability of an event happening = *Number of ways it can happen* **Total number of outcomes**

Example: there are 4 Kings in a deck of 52 cards. What is the probability of picking a King?

Number of ways it can happen: 4 (there are 4 Kings)

Total number of outcomes: 52 (there are 52 cards in total)

So the probability = *4* **52** = *1* **13**

Mutually Exclusive

When two events (call them "A" and "B") are Mutually Exclusive it is **impossible** for them to happen together:

P(A and B) = 0

"The probability of A and B together equals 0 (impossible)"

But the probability of A **or** B is the sum of the individual probabilities:

P(A or B) = P(A) + P(B)

*"The probability of A **or** B equals the probability of A **plus** the probability of B"*

Example: A Deck of Cards

In a Deck of 52 Cards:

- the probability of a King is 1/13, so **P(King)=1/13**
- the probability of an Ace is also 1/13, so **P(Ace)=1/13**

When we combine those two Events:

- The probability of a card being a King **and** an Ace is **0** (Impossible)
- The probability of a card being a King **or** an Ace is (1/13) + (1/13) = **2/13**

Which is written like this:

P(King and Ace) = 0

P(King or Ace) = (1/13) + (1/13) = 2/13

Special Notation

Instead of "and" you will often see the symbol ∩ (which is the "Intersection" symbol used in Venn Diagrams)

Instead of "or" you will often see the symbol ∪ (the "Union" symbol)

Example: Scoring Goals

If the probability of:

- scoring no goals (Event "A") is **20%**
- scoring exactly 1 goal (Event "B") is **15%**

Then:

- The probability of scoring no goals **and** 1 goal is **0** (Impossible)
- The probability of scoring no goals **or** 1 goal is 20% + 15% = **35%**

Which is written:

P(A ∩ B) = 0

P(A ∪ B) = 20% + 15% = 35%

Remembering

To help you remember, think:

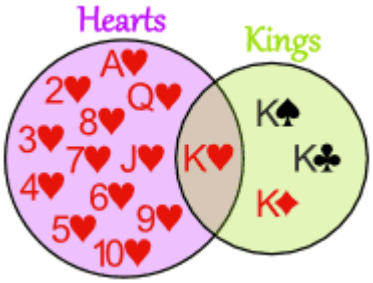
"**Or** has **more** ... **than And**"

Also ∪ is like a cup which holds **more** than ∩

Not Mutually Exclusive

Now let's see what happens when events are **not Mutually Exclusive**.

Example: Hearts and Kings



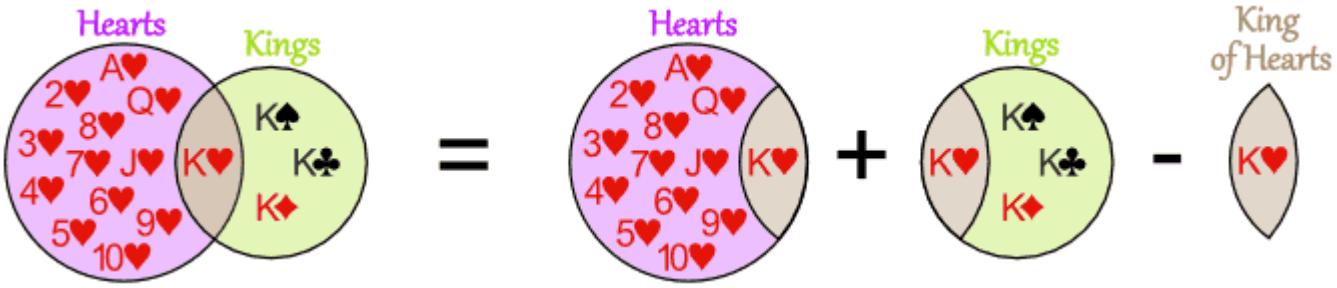
Hearts **and** Kings together is only the King of Hearts: 

But Hearts **or** Kings is:

- all the Hearts (13 of them)
- all the Kings (4 of them)

But that counts the King of Hearts twice!

So we correct our answer, by subtracting the extra "and" part:



16 Cards = 13 Hearts + 4 Kings − the 1 extra King of Hearts

Count them to make sure this works!

As a formula this is:

P(A or B) = P(A) + P(B) − P(A and B)

*"The probability of A **or** B equals the probability of A **plus** the probability of B **minus** the probability of A **and** B"*

Here is the **same formula**, but using ∪ and ∩:

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

A Final Example

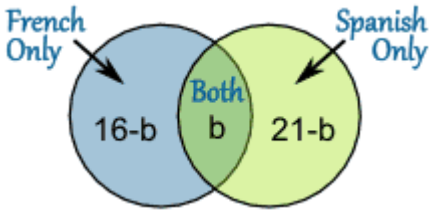
16 people study French, 21 study Spanish and there are 30 altogether. Work out the probabilities!

This is definitely a case of **not** Mutually Exclusive (you can study French AND Spanish).

Let's say **b** is how many study both languages:

- people studying French Only must be 16-b
- people studying Spanish Only must be 21-b

And we get:



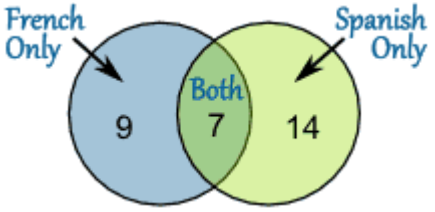
And we know there are **30** people, so:

$(16-b) + b + (21-b) = 30$

$37 - b = 30$

$b = 7$

And we can put in the correct numbers:



So we know all this now:

- P(French) = 16/30
- P(Spanish) = 21/30
- P(French Only) = 9/30
- P(Spanish Only) = 14/30
- P(French or Spanish) = 30/30 = 1
- P(French and Spanish) = 7/30

Lastly, let's check with our formula:

P(A or B) = P(A) + P(B) − P(A and B)

Put the values in:

30/30 = 16/30 + 21/30 − 7/30

Yes, it works!

Summary:

Mutually Exclusive

- A **and** B together is impossible: **P(A and B) = 0**
- A **or** B is the sum of A and B: **P(A or B) = P(A) + P(B)**

Not Mutually Exclusive

- A **or** B is the sum of A and B minus A **and** B: **P(A or B) = P(A) + P(B) – P(A and B)**